Nagelian Reduction Beyond the Nagel-Model

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Abstract: Nagel’s official model of theory-reduction and the way it is represented in the literature are shown to be incompatible with the careful remarks on the notion of reduction Nagel gave while developing his model. Based on these remarks, an alternative model is outlined which does not face some of the problems the official model faces. Taking the context in which Nagel developed his model into account, it is shown that the way Nagel shaped his model and, thus, its well-known deficiencies, are best conceived of as a mere by-product of his philosophical background.
Introduction

Here is an upshot of how Nagel defines the notion of reduction:

[A] reduction is effected when the experimental laws of the secondary science (and if it has an adequate theory, its theory as well) are shown to be the logical consequences of the theoretical assumptions (inclusive of the coordinating definitions) of the primary science. (Nagel 1961, p. 352)

The basic idea is simple: A theory $T_R$ reduces to a theory $T_B$ iff $T_R$ is derivable from $T_B$ plus the relevant bridge laws (here labeled `coordinating definitions`), if any, with the contention that, often, theory reduction is carried out by reduction of the theory’s laws. If we add the remarks Nagel opened his discussion on reduction with – namely, that reduction has to be understood as a certain kind of explanation (Nagel 1961, 338) – the core idea of the Nagel model is fully characterized. Reduction is (i) a kind of explanation relation, which (ii) holds between two theories iff (iii) one of these theories is derivable from the other, under some conditions with (iv) the help of bridge-laws. This is what is referred to as the Nagel model of reduction in the philosophical literature. It covers two versions of reduction, depending on whether or not bridge-laws are required: non-homogeneous cases and homogeneous cases. Nagel conceives of sciences or theories as developing entities which undergo changes, even though the vocabulary remains unchanged (but is, presumably, often extended). These successive states of theories are covered by the notion of homogeneous reductions – deduction of an early stage from a later stage of a theory is possible without bridge-laws. Non-homogeneous cases of reduction are instantiated by pairs of different theories, employing different vocabularies. Whereas the former variant of reduction did not attract much attention (by Nagel and others), the latter has been intensively discussed ever since Nagel introduced his model in
“The Meaning of Reduction in Natural Science” (Nagel 1949). Accordingly, this paper’s focus lies on non-homogeneous reductions.

Based on more elaborate versions of the rough outline just given, the Nagel-model became subject to a number of well known criticisms:

It is too narrow because it allows only for theory reduction (Sarkar 1992; Darden & Maull 1977, 43; Hull 1974; Wimsatt 1974), whereas an appropriate model would cover cases of reduction of mere models and the like – sciences like biology and neuroscience should be regarded as being possible candidates for reduction, although they do not contain full-fledged theories.

It exemplifies all the shortcomings of the orthodox view on science. For example, it conceives of theories as syntactic entities and of explanation to be cashed out in terms of the DN-model (Hempel & Oppenheim 1948), which has been criticized for a number of reasons, amongst which issues regarding the asymmetry of explanation play a distinguished role (for an overview which focuses on problems arising from reduction as explanation, see Craver 2007, chapter 2, for problems concerning the DN model, see Salmon (1989)).

It describes reduction in terms of direct theory-explanation, whereas an appropriate model of reduction should shape the notion in terms of indirect theory-explanation, that is: in terms of explanation of the phenomena of a theory (Schaffner 1967; Kemeny & Oppenheim 1956; Friedman 1982).
In addition, there are at least three formal worries worth mentioning: If reduction is
derivation plus (sometimes) bridge-laws, then any theory would reduce to itself (because
any theory is derivable from itself); moreover, any theory would reduce to any
inconsistent theory; and contrary to what one might expect, reduction is not an
asymmetric relation – derivability does not amount to asymmetry. So, the Nagel-model
seems to be in a very bad shape.

Despite these alleged problems, Ernest Nagel’s model of theory-reduction shaped the
discussion on reduction in the philosophy of science and it received considerable
attention in the philosophy of mind (see, for example, Kim (1993, 150 & 248) and Fodor
(1981, 150)). More recent approaches to reduction depart from the Nagel model (Dizadji-
Bahmani et al, 2010; Bickle 1998 & 2003; Schaffner 1993; Churchland 1985, Hooker
1981), and it has been argued that most of these approaches merely echo the Nagel-model
instead of proposing fundamentally new interpretations (Endicott 1998 & 2001; Dizadji-
Bahmani et al. 2010). Thus, an appropriate characterization of the model Nagel proposed
may (in addition to its historical value) function as a substantial contribution to the
current debate on reduction. Unfortunately, an appropriate characterization of Nagel’s
model has not been given yet – the ways in which the model is usually described do not
capture the rich descriptions of the notion of reduction Nagel offered (Nagel 1935, 1949,
1961 chapter 11 & 1970). The aim of this paper is to fill in this gap contrasting the way
Nagel’s model is described in the literature with what I believe to be a more appropriate

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1 Let me briefly comment upon the four main publications of Nagel’s which are directly concerned with
reduction. In his 1935, Nagel gives an intuitive sketch of reduction which is almost fully lost in his later
writings, but which is interesting for some interpretative purposes picked up in the present paper. His 1949
paper is almost fully included in chapter 11 of his 1961 book, which includes some extensions, the most
important of which will be discussed below. The point of departure for his interpreters is chapter 11 of his
(1961), so I will mainly focus on this chapter. His 1970 paper is interesting in three respects: First, it includes
an interpretation of bridge-laws which is much more precise than the interpretation offered in (Nagel 1961),
second, it includes a discussion of the role of correction in reduction, and it sheds light onto issues regarding
the question of the relata of the reduction relation.
and more powerful characterization of the model. It will turn out that most of the misunderstandings have their routes in the way Nagel himself presented his model.

The Nagel model as described in the literature will be referred to as the official Nagel model – it is a mixtum compositum of parts of the definition Nagel offered and ways of representing Nagel’s definition which are common among his interpreters. Having sketched the official model in section 1, it will then be contrasted with what I believe to be the real Nagel-model in section 2. In section 3, it will be shown that the real Nagel-model is more potent than the official model, and an explanation for why Nagel’s official model differs from the real one is offered.

1. The official Nagel-Model

In order to sketch the official Nagel-model, we should reflect upon the four aspects obtained from the definition given above: Reduction is (i) an explanation of (ii) theories, which is cashed out in terms of (iii) derivation with (sometimes) (iv) the help of bridge-laws. According to these four main features of the official model, there are four fields of additional questions which need to be answered to fully characterize the model. Such questions regard:

(i) Reduction as explanation.

(ii) The relata of the reduction relation.

(iii) Derivability.
(iv) The status of bridge-laws.

Philosophers have commented extensively upon these four aspects of the Nagelian definition, and for any single one of them, we can distinguish between several fields of sub-questions (for example, the problem of bridge-laws gave rise to logical, epistemological and ontological considerations.)

Let us consider these aspects in the order just given. The body of suggestions to be assembled here does not have to be conceived of as a coherent or exhaustive picture of how the Nagel-model is represented in the literature – there is no such picture. Rather, we will develop an idealized version of the family of models which figure under the title ‘Nagel-reduction’. The model so developed will be the subject of a critical examination in section 2.

1.1 Reduction as Explanation

Let us begin with a discussion of considerations regarding the Nagel model as a model of explanatory reduction. According to Sarkar (1992, 173) we should read Nagel as proposing a non-ontological, but rather an epistemological model of reduction. Unfortunately, Sarkar is not very clear about what he takes to be the difference between ontology and epistemology. The Nagel model is judged a “purely epistemological issue with no necessary ontological commitment” (Sarkar 1992, 173) because the model is regarded as a model of explanation. The crucial premise is that “[e]xplanatory reductionism is epistemological reductionism” (Sarkar 1992, 171). So, the argument
Sarkar puts forward is this: According to the Nagel-model, reduction is a specific kind of explanation. Therefore, it is a purely epistemological model. Here, we have our first characterization:

*The Nagel-model is an epistemological model of reduction.*

Independent of considerations concerning explanation, the claim that the Nagel-model is an epistemological model of reduction has two other sources:

(i) Nagel’s model is judged to be the “well known logical-empiricist explication of epistemological reductionism” by Hoynigen-Huene (1989, 30). This claim is based on another criterion for being epistemological, which is nicely captured in Silberstein (2002, 80f.) and also pertinent in Sarkar (1992): The model is judged *epistemological* because it is concerned with *theory*-relations.

(ii) Peter Fazekas (2009), following (Klein 2009), argues that according to Nagel, bridge laws are to be described in purely epistemological terms (the reason for why they do this will become apparent in section 1.4). Again, this gives rise to an epistemological characterization of the official Nagel-model.

Another aspect of the official model which is closely tied to explanation is captured by the idea that it is a model of *direct* reduction. Clearly, the model invites us to conceive of the explanation relation as holding *directly* between theories. Therefore, the Nagel model is sometimes (*cf.* Schaffner 1967, 137 & 1993, 423) described as follows:

*Nagel-reduction amounts to direct reduction.*

This is contrasted with the view that there are *indirect* reductions – reductions which hold in virtue of the fact that the reducing theory explains the occurrence of the phenomena of
the reduced theory (rather than the reduced theory itself) – a view associated with Kemeny & Oppenheim (1956) and Friedman (1982). Let us try to make this idea more precise: A definition of an indirect reduction-relation has (among other features) to define an n-place-predicate, two arguments of which are terms referring to theories (or some other representational devices), and one argument of which is a term referring to the phenomena of the reduced theory, such that the reducing theory explains these phenomena. Models of direct reduction lack this latter feature. In this sense of the distinction between direct and indirect reduction, the Nagel-model is interpreted as a model of direct reduction. This brings us to our next point.

1.2 The relata of the reduction relation

Especially among modern philosophers of science who are skeptical about the possibility of reconstructing all scientific representations as full-fledged theories, it is common to argue that Nagel’s model of reduction is misguided because it takes reductions to connect full-fledged theories, rather than models, (mere) descriptions, fragments of theories or facts (see, for example: Sarkar 1992 &1998; Darden & Maull 1977, 43; Hull 1974; Wimsatt 1974). So, we face the following problem:

Reduction is a relation holding between theories.

The relevant relation is, as already said, explanation, which is cashed out in terms of derivability.

1.3 Derivability
Independent of whether or not derivability is an appropriate model for explanation or reduction, we should note that philosophers agree that

Nagel-reduction is derivation plus bridge-laws.

And rightly so! At least, this is what Nagel himself tells us. (That this is not the whole story yet will be argued for below). In contrast to this simple matter, bridge-laws form a much debated issue.

1.4 The status of bridge-laws

Nagel (Nagel 1961, chapter 11, section II. 3) discusses three different candidates for the “nature of the linkages postulated” (Nagel 1961, 354) by bridge laws:

alternative 1: The links mimicked by the bridge-laws are “logical connections” (Nagel 1961, 354), which are understood as meaning connections.

alternative 2: The links mimicked by the bridge-laws are “conventions” (Nagel 1961, 354), or stipulations (“deliberate fiat” Nagel 1961, 354).

alternative 3: The links mimicked by the bridge-laws are “factual or material” (Nagel 1961, 354), that is, bridge-laws state empirical facts (these truths are described as hypotheses).

It is worth noting that these alternatives give semantic characterizations of bridge-laws. It is the links which are “postulated” by bridge-laws which are characterized here. A characterization of bridge-laws as syntactic entities (as discussed below, see section 2.3)
which is required for an interpretation of bridge-laws as functioning in (syntactic) derivations would be derivative upon this characterization.

So, what do these links consist in? Alternative 1 takes the link to be a conceptual one, thus presupposing that the entities so linked belong to the realm of concepts. At first sight, alternative 2 seems to take the entities linked to be expressions, because it is expressions we can be linked by convention. Alternative 3 speaks, somewhat underdetermined, about factual or material links. Accordingly, what is linked seems to be some worldly entities, like properties or states of affairs. Thus, here we have different kinds of semantic characterizations. The last differs from the first in that it is concerned with the non-conceptual aspects of meaning, whereas the first is concerned with conceptual aspects of meaning. How alternative 2 fits into the picture of semantic characterizations will be discussed below.

It seems that in addition to the semantic characterizations, an epistemological aspect is crucial for an appropriate understanding of these three alternatives (this is what Fazekas (2009) seems to focus on): According to alternative 1, we know about the truth or falsity of a bridge law in virtue of grasping a meaning already established. According to alternative 2, bridge-laws have the epistemological status of a convention. And according to alternative 3, we know about the truth or falsity of a bridge law in virtue of empirical investigation. This interpretation perfectly matches the additional characterization of the last alternative as committing us to the claim that bridge-laws are hypotheses (Nagel 1961, 354). Thereby, Nagel describes a relevant epistemological consequence of the semantic characterization of bridge-laws as stating factual or material connections – what

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2 Interestingly, in his 1949 paper Nagel does not discuss this second option. In his 1970, he seems to deny that it is a serious option, arguing that bridge laws are not “arbitrary stipulations” but rather empirical hypotheses (1970, 126).
he labels the “cognitive status” of bridge-laws (Nagel 1961, 356). So, Nagel gives us the candidates for an appropriate interpretation of bridge-laws in terms of the semantic/epistemological status they have. Let us now turn to the question of how Nagel judges these candidates.

Arguing that we should not regard, for example, ‘temperature’ as having the same meaning as ‘mean kinetic energy’ (nor to take the meaning of the latter to be analytically derivable from the meaning of the former), Nagel argues that we can easily see why the first alternative fails to adequately describe the nature of bridge-laws: There just is no analytic connection which would allow the bridge-principles to be true or knowable in virtue of meaning. Interestingly, Nagel does not dismiss alternative 2 for the same reason. He describes stipulations as “conventions […] [which] institute a correspondence between” the two expressions so linked (Nagel 1961, 354). Note that therefore, the convention mentioned in alternative 2 should not be regarded as a linguistic convention, fixing the use, and, if you like, the meaning by stipulation. Otherwise, Nagel could have dismissed this alternative on the same grounds on which he dismissed the first alternative: If someone utters “Something is temperature (has temperature value n) iff it is mean kinetic energy (has mean kinetic energy value n*, with n being proportional on n*)” in the context of reduction, we can be sure that this statement is not analytic. Thus, it cannot be an instance of a linguistic convention. So, whatever Nagel took these conventions to be, he did not believe them to be meaning stipulations (we will turn back to this point in a moment, see section 2.3).

Nagel did not decide the issue between the knowable-by-convention and the knowable-by-investigation-of-empirical-fact variant, at least in the section just referred to. Accordingly, he is presented in the literature as embracing the following claim (to be
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understood as follows: Nagel explicitly leaves the question of whether bridge-laws are to be regarded as stipulations or as empirical hypotheses open.) (for example, Schaffner 1967, 145):

*Bridge-laws are to be regarded as stipulations or as empirical hypotheses.*

Thus, we have assembled the most common characterizations of Nagel’s model of reduction and are now in a position to summarize the official Nagel-model (let me change the order of these aspects, because it will enable us to structure the discussion in the next section):

a. Reduction is a relation holding between theories.

b. Reduction is derivation plus bridge-laws.

c. Bridge-laws are to be regarded as stipulations or as empirical hypotheses.

d. Reduction is direct.

e. The Nagel-model is an epistemological model of reduction.

In the next section, I shall go through these characterizations and argue that they all fail to represent the core idea underlying the Nagel-model appropriately, some because they are incomplete, others because they are plainly false.

2. The real Nagel-model
It will now be shown that there are tensions between the official Nagel-model (as described by Nagel and others) and the non-technical or explicatory remarks concerning this model offered by Nagel. Since the way the official model is described gives rise to a number of problems mentioned in the introduction and during the discussion in section 1, we should take these latter remarks as serious candidates for a refined model, which is maybe able to deal with some of these problems.

Before we start, here is a brief note concerning the exposition of this section. The interpretations I propose are arranged such that they match the features of what I have coined the ‘official Nagel model’. Some of these features require brief comments only whereas others deserve more careful investigation, which enables us to introduce further ideas pertinent in Nagel and lost during the discussion. So, we go through the five features of the official Nagel model ((a) to (e)). Before turning to the next feature, the feature discussed will be transformed into a characterization of the same issue which comes much closer to what can be extracted from Nagel’s informal remarks on reduction.

Here is the first feature of the official Nagel-model: a. Reduction is a relation holding between theories.

2.1 Reduction as a relation holding between theories

According to the official model, the relata of the reduction relation are (necessarily) theories. Some philosophers are worried about this interpretation because the notion of a theory Nagel employed is quite demanding: Psychology, neuroscience and parts of biology would not turn out to be theories according to Nagelian standards (see Nagel 1961, chapter 11). However, there is an important point to be made. Nagel chose to define
reduction as a relation holding between theories in order to define epistemically ideal (but not the only possible) cases of reduction. Let me explain.

Nagel maintained that his “requirement of explicitness”, which allows for reductions only in cases of theories of which the laws, axioms etc. are explicitly stated, was “an ideal demand, rather than a description of the actual state of affairs that obtains at a given time” (Nagel 1961, 345). Interestingly, this requirement is to be understood as an epistemologically justified ideal, concerning the justification of reductions. The argument he gives in favor of this requirement is this: “To the extent that this elementary requirement is not satisfied, it is hardly possible to decide with assurance whether one science (or branch of science) has in fact been reduced to another” (Nagel 1961, 345, emphasis added). On the same occasion, Nagel acknowledges that this criterion of maximal explicitness is only in a few, if any cases “fully realized, since in the normal practice of science it is rarely necessary to spell out in detail all the assumptions that may be involved in attacking a concrete problem” (Nagel 1961, 345). So, according to Nagel, reduction of models, fragments of theories, isolated statements and so forth is not excluded in principle, although it may pose epistemological problems.

That Nagel indeed was more liberal about the relata of reduction can easily be seen once we consider his comments on the reducibility of biology (1970, 123, & section IV): In its present state, we do not see at all how a reduction of biology could be effected, although “some processes occurring in living organisms can now be understood in terms of physicochemical theory” (Nagel 1970, 123) – and fragments of biology can thus be reduced without these fragments instantiating Nagel’s demanding notion of a theory. Thus, although Nagel uses the notion of a theory in his definition, he allows for other sorts of reduction. He just assumes that the only cases of reductions we can know about
“with assurance” are cases of theory-reductions. The epistemological status of other reductions is weaker.

Nevertheless, as Nagel’s discussion of the idea of property-reduction shows (1970, 119; 1961, section 11, III, 3), he seems to embrace the claim that the primary relata of the reduction-relation be representational in nature, rather than ontological (for a more detailed discussion of the same idea, see (Hempel 1965, chapter 8 & 1969)). In the relevant sections Nagel argues that any notion of property-reduction is dependent upon reduction of representational items, because properties are not directly available in discourse, but are, instead, presented by the relevant representational items which are, then, subject to operations like explanation and derivation. Thus, it should be noted that even though Nagel is more liberal about what sort of representational devices can enter the reduction-relation than he is actually described, he nevertheless assumes that talk about property reduction independent of reduction of representational entities (like theories) is misguided.

In summary, according to Nagel, reduction is not a relation holding between theories only, although theory-reductions play a distinguished epistemological role. Nevertheless, sciences like neuroscience and parts of biology are covered by the definition. Therefore, we should replace a. by a.*:

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a.* \text{ Reduction is a relation holding among a great variety of scientific representational devices, amongst which theories play a prominent epistemological role.}
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As has been pointed out by some (Sarkar 2008, 426f.; Wimsatt 1976) and is still neglected by others, reduction according to Nagel is often more than derivation plus
bridge-laws. So, let us turn to the second feature of the official Nagel-model: b. Reduction is derivation plus bridge-laws.

2.2 Reduction as derivation plus bridge-laws

Can reduction be fully characterized in terms of derivation (given we have the appropriate bridge-laws)? That depends. According to Nagel, we have to distinguish between non-trivial and trivial reductions (Nagel 1961, 358). This distinction is defined by a set of “nonformal conditions” for reduction, which have to be fulfilled for a reduction to be non-trivial (Nagel 1961, 358 ff & 1949, 304 ff.). These criteria have gained little attention in the literature, even though refined versions of the definition of the concept of reduction explicitly include criteria similar to those proposed by Nagel (Schaffner 1993; Churchland 1986). But why do we need additional criteria to distinguish non-trivial from trivial cases of reduction? For example, we could just construct a theory from which thermodynamics is derivable (with or without bridge-laws), but which is complete nonsense. This is what Nagel seems to have had in mind when he wrote:

If the sole requirement for reduction were that the secondary science is logically deducible from arbitrarily chosen premises, the requirement could be satisfied with relatively little difficulty. (Nagel 1961, 358)

So, how to exclude such cases? In section 11.III (Nagel 1961) Nagel introduces four nonformal criteria for reduction.3

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3 The section is somewhat unstructured. Only one third is concerned with non-formal criteria, the other two thirds being about a temporal understanding of theories and the question of whether or not properties can be said to reduce.
The first criterion Nagel mentions is that the premises – the bridge-laws and the reducing theory – from which the reduced science is deduced should be well established rather than arbitrarily chosen (Nagel 1961, 358). (Just as an aside: This seems to be incompatible with the view that bridge-laws are conventions. Conventions are never well established in the sense relevant here.)

Secondly, Nagel alludes to the fact that the reducing theory should be better established than the reduced one. Referring to the kinetic theory of gases, he argues that the evidence for this theory comes from a wider range of phenomena than the evidence for thermodynamics (Nagel 1961, 358) – just because it is concerned with more sorts of phenomena.

As a third criterion, we find the idea that reduction is concerned with unification: The underlying theory unifies laws which seemed unconnected before the actual reduction took place (Nagel 1961, 359). According to Nagel, the result of unification is an epistemological strengthening of the reduced science.

The fourth criterion is this: The reducing theory corrects and augments the reduced theory. This is astonishing in at least one respect: It anticipates what has by some (Feyerabend 1962 & 1966; Schaffner 1993; Bickle 1998; Churchland 1986) been regarded as figuring among the most threatening counterarguments to Nagelian reduction; that it neither allows for correction in the reducing theory, nor for replacement. Nagel writes:

The [reducing] theory must also be fertile in usable suggestions for developing the secondary science, and must yield theorems referring to the latter’s subject matter which augment or correct its currently accepted body of laws. (Nagel 1961, 360)
At first sight, it is not clear how correction is possible if deduction is the basis for reduction. The truth of the reducing theories grants the truth of the reduced theories. This is what deduction is about. So, it is worth noting that Nagel introduced this criterion, even though it seems hardly compatible with his official model. However, as suggested by Putnam (1965, 206ff, esp. footnote 3), there are a number of ways to incorporate correction within Nagel’s model: by speaking of approximate truth, by contextualizing the reduced theory, or by introducing probabilistic notions – a strategy which anticipates models of New-Wave reduction as proposed, for example, by Schaffner (1967 & 1993), Hooker (1981) and Bickle (1998). Moreover, and, for our present concern, more importantly, Nagel, discussing Feyerabend’s criticism, explicitly states that reductions may rely on approximations (1970, 120ff. & 133). A similar idea seems to be tied to Nagel’s suggestion that sometimes, boundary conditions might be necessary to effect a reduction (Nagel 1961, 434) – namely, in order to connect the relevant kind terms in an appropriate bridge-law, thereby altering and adjusting one of the terms’ extensions. Thus, it seems plausible to assume that Nagel believed correction to be possible in reduction.

We have assembled four additional criteria which distinguish non-trivial or interesting reductions from trivial ones. These must not be overlooked when we try to evaluate the Nagelian approach to reduction:

b.* **Interesting reductions are explanations which consist in deductions which are carried out with the help of bridge laws, and reductions of this sort have to obey (some of) the non-formal criteria just given.**

Note that if these criteria are incorporated in the definition (or if they are captured by some other formal criteria), we do not have a relation holding between theories and bridge-principles only: The property of being well established and the property of being
better established than something else are relational properties in the sense that nothing is well established \textit{tout court}. These relations hold among theories and \textit{sets of phenomena} with respect to which these theories are established. Reduction as a relation holding between theories (and bridge-laws) only does not sufficiently characterize the interesting, non-trivial cases of reduction. However, the characterization of the Nagel-model as being only concerned with theories (and bridge-laws) was misguided from the beginning. In order to show this let us consider the third and the fourth feature of the official Nagel-model, which will bring us back to questions regarding bridge-laws, and theory-reduction as theory-explanation.

Here is the \textit{third} feature of the official Nagel-model: \textit{c. Bridge-laws are to be regarded as stipulations or as empirical hypotheses.}

\subsection*{2.3 Bridge-laws as stipulations and empirical hypotheses}

Bridge laws can be classified according to different categories. The classification Nagel suggests in (Nagel 1961, 353 ff) is a classification with respect to certain semantic and epistemological aspects. In addition, we find syntactic and ontological characterizations in his writings. Since these characterizations depend upon different distinctions which are normally not reflected in the literature on reduction, and since they enable us to discuss the status of bridge-laws in general, a careful investigation of these rival descriptions is in place. It will turn out that merely some of these characterizations are compatible. Accordingly, we will have to choose between the rival candidates.
The most common way to describe bridge laws syntactically is in accordance with the official Nagel model (section 11, II, 3 of his (1961)): Bridge laws are bi-conditionals.\(^4\) For example, the pair ‘\_is\_/has a certain mean kinetic energy’ and ‘\_is\_/has a certain temperature’ is connected by a bridge-law of the form ‘\(\forall x, Fx \leftrightarrow Gx\)’. Trouble starts when we reconsider the claim of Nagel’s about the relation of postulation allegedly holding between bridge-laws and the relevant semantic/epistemic links. Obviously, there is a huge gap between characterizing a sentence as a bi-conditional and as postulating the relevant sort of link. The only link bi-conditionals can be used to state is a semantic link fully determined in terms of dependence of truth-values – that’s it. Thus, the syntactic characterizations can only be characterizations of dummies for real bridge-laws within a formal model. Bi-conditionals are rendered true by a great variety of “links” (but being rendered true by is not to be conflated with postulating): contingent co-instantiation or co-occurrence is not enough for reduction even though it is enough for some bi-conditionals to hold (and, therefore, for some pairs of (possible) theories to instantiate the official Nagel-model). Therefore, logical form (in first order predicate logic)\(^5\) is not what we should focus on when we consider the syntactic nature of bridge-laws. But what, then, is the relevant characterization?

On several occasions, Nagel characterizes bridge-laws according to categories much finer-grained than the ones presented so far. He argues that when we discover that some science reduces to another science, we discover that the traits of objects which are genuine to the reduced science are in fact identical to traits the reducing science deals with (Nagel 1961, 340). On another occasion, Nagel describes the reduced science’s

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\(^4\) In a footnote, Nagel troubles his interpreters with the suggestion that bridge-laws may take the form of conditionals. What holds for bi-conditionals holds a fortiori for conditionals. Thus, I shall focus on the latter.

\(^5\) Dizadj-Nahmani et al. (2010) conclusively argue that for a model of reduction to be Nagelian in spirit, it does not have to rely on first order predicate logic.
predicates as being (in principle) *definable* in terms of the reducing science (Nagel 1961, 434), and definability as necessary for the construction of bridge-laws. Since it is plausible to assume that the open sentences which form the definiens and the definiendum of an appropriate definition pick out the same property, we have, again, property-identity. Similarly, in his (1970), Nagel explicitly states that under one relevant interpretation of the reduction relation, bridge-laws state identities among properties – here labeled 'attributes' (1970, 127f.). On the same occasion, he discusses relations among the relevant terms’ *extensions* as alternative interpretations (1970, 126f.). Two candidates are worth mentioning: (i) co-extensionality of the terms and (ii) the relation of one term’s extension being included in the other term’s extension. Since, again, pure coincidence of co-instantiation should be ruled out – because it would not be sufficient for reduction – nomological necessity is required to fully characterize the relevant connection. That nomological necessity should be incorporated in the characterization is also suggested by Nagel’s discussion of examples (temperature and kinetic energy and the partial dependence of colors upon physical optics). Here is the list of rival characterizations we thus obtain:

Semantic/Ontological/Epistemic:

(i) Identity

(ii) Relation among extensions

(iii) Stipulation (convention)

(iv) Hypotheses (factual or material)

(v) Conceptual connection (logical)
Syntactic:

(vi) Bi-conditional

Let us resume the discussion of these candidates to get a picture of how bridge-laws should be conceived. As already stated, the syntactic characterization Nagel officially considers has to be dismissed – it does not capture the relevant semantic/epistemic characterizations.

The interpretation of bridge-laws as conventions or stipulations is not very clear. As already mentioned, it cannot be meaning convention, because if it was, it would have to be dismissed on the same basis as the interpretation of bridge-laws as analytic truths had been dismissed. Secondly, the idea that bridge-laws are conventions does not fit the idea that bridge laws must not be arbitrary, but rather be well established (this seems to be the interpretation of Dizadji-Bahmani et al. (2010, section 4)). But we do get some hints leading towards what Nagel had in mind in the discussion of candidates (ii) and (iii) (stipulation and hypotheses). During this discussion, Nagel focuses on purely cognitive aspects, which give rise to a different interpretation of (ii). The criterion to distinguish between (ii) and (iii) can be put as follows: We have a case of (ii) iff the claim made by the relevant statement is not based on empirical data and is not based on meanings. We have a case of (iii) iff the claim made by the relevant statement is based on some empirical data. So, these sorts of statements are to be distinguished with respect to how they are justified in a certain context (Nagel 1961, 356). Thus, the term ‘stipulation’ should not be taken literally – instead, we could describe these cases as tentative assumptions which are epistemologically weaker than empirically justified (though not reliably tested) hypotheses. When we connect mean kinetic energy to temperature, we make it available for measurement (Nagel’s example). However, we do not thereby
stipulate that mean kinetic energy is temperature (no one could do that), even though in this context, we cannot test the assumption independently, just because the only test available depends on this very assumption. But this characterization does not give us an idea of how to conceive of the relevant link. Again, mere coincidence is not ruled out, and hypotheses and stipulations may just be false. Just like taking the form of bi-conditionals, being an empirical hypothesis is underdetermined with respect to how the link should be conceived of in order to yield reductions.

In his (1961), Nagel considers just one candidate which meets this requirement: identity. In his (1970), he repeats this idea and in addition considers relations holding among the relevant terms’ extensions. Since these are the only serious candidates for a complete characterization of bridge-laws, we should interpret Nagel as taking bridge laws to state identities or nomologically necessary relations among extensions. In addition, bridge-laws are, according to Nagel’s characterization as stipulations (in the Nagelian sense) or hypotheses, a posteriori in the Kripkean sense (Kripke 1980). This suggests another logical form for bridge-laws, now connecting terms (`A=B` and `nom the extension of A= the extension of B`). These terms refer to properties or natural kinds or their extensions, and they have different meanings (if any). We thus get:

\[ c.~^*~ Bridge-laws~are~to~be~regarded~as~stating~ontological~links~(identities~or~relations~among~extensions)~a~posteriori. \]

Now, if bridge-laws should be conceived of as stating identities or relations among the relevant terms’ extensions, then clearly, reduction incorporates reference to the theories’ ontologies, such that reduction is, according to this view, more complex than any two-place relation holding among theories only. Consider the following passage:
In [...] cases [of reduction in which the vocabularies of the two \textit{relata} of the reduction relation differ, \textit{RvR}], the distinctive traits that are the subject matter of the [reduced] science fall into the province of a theory that may have been initially designed for handling qualitatively different materials [...]. The [reducing] science thus seems to wipe out familiar distinctions as spurious, and appears to maintain that what are prima facie indisputably different traits of things are really identical. (Nagel 1961, 340)

On this occasion, Nagel describes reduction as showing seemingly distinct \textit{traits} to be \textit{identical}. This stands in stark contrast to the focus on theories in the definition. The purpose of bridge-principles can, in this light, be described as contributing the relevant ontological meat to the skin-and-bone model of theories. Bridge-laws cannot be characterized in purely syntactic terms. They are individuated by what their kind-terms refer to, or by what their predicates signify.

How this implicit reference to the theories’ ontologies is to be interpreted and how identity and bridge-laws fit into the picture of reduction as explanation will be considered in the next step.

Here is the \textit{fourth} feature of the official Nagel-model: \textit{d. Reduction is direct.}

\subsection*{2.4 Direct and indirect reduction}

This claim is of utmost importance because it brings us to the question of the role explanation plays within the Nagelian model of reduction. Let us turn to the description of reduction Nagel opened the discussion with:
Reduction, in the sense in which the word is here employed, is the explanation of a theory or a set of experimental laws established in one area of inquiry, by a theory usually though not invariably formulated for some other domain. (Nagel 1961, 338)

For Nagel, the question of what ‘explanation’ means here is settled: It is explanation according to the DN-Model (Hempel & Oppenheim 1948). Accordingly, Nagel conceives of explaining one theory by another theory in terms of deduction. But why does he believe that reduction is explanation of theories? Focusing on this question, we will be able to deliver the resources needed to judge the issues (i) of reduction as theory-explanation and (ii) of direct vs. indirect reduction. It will turn out that it is extremely difficult to make sense of the idea that the reducing theory explains the reduced theory within the framework of explanation Nagel had in mind. It will be concluded that reduction is neither theory-explanation (in this specific sense) nor direct. Fortunately, and contrary to what one might expect given the official model, this is in accordance with Nagel’s own intuitions. In this context, it worth noting that Nagel never argued for the claim that reduction is theory-explanation (and rightly so, because such an attempt would be futile). He just introduced his model as a model of theory-explanation. As we shall see in a moment, what he actually described was another sort of explanation which amounts to indirect reduction.

If reduction is explanation in the Nagelian sense, then a theory can explain another theory. This is to be interpreted in a strict sense: The relata of the reduction relation are the relata of the corresponding explanation. In the Nagelian sense, theories are subject to procedures such as derivation, representation and the like, thus: They are linguistic entities. There is a straightforward way to show that theory-reduction cannot be theory explanation simpliciter. Let us take theories to be structures of statements. Do statements explain other statements? In one sense, they do not, and cannot, namely, when we take
statements to be non-interpreted sequences of signs. Therefore, we should interpret theories as being interpreted linguistic entities. Then one might want to hold that in a sense, ‘there was an earthquake’ explains ‘some windows shattered’ (conceived of as meaningful sentences of English), if an earthquake made some windows shatter. Let me firstly point to a logical problem this characterization faces, and then move on to an interpretation of how such explanations might turn out appropriate (given that the logical problem could be ignored). Based on this interpretation, it can be shown that direct theory explanation is derivative upon indirect theory explanation.

Here is the logical problem: Even though the predicate ‘_ explains _’ takes singular terms as arguments, it seems that it does not take singular terms which refer to linguistic entities. Rather, it takes terms referring to events (the occurrence of the earthquake explains the shattering of the windows), or terms referring to propositions or states of affairs (that the earthquake occurred explains why the windows shattered) as arguments. In non-scientific contexts, it might take a term which refers to a linguistic entity on the right hand side: We can explain the meaning of a linguistic object, or its grammatical structure, but we do so by using a statement rather than mentioning one. The same holds for cases of explaining a given theory (as a linguistic entity) to a student. We do so giving meaning explanations, maybe referring to the historical context in which the theory was developed, explain syntactic aspects and so forth. However, it is not easy to see how we could construct an instance of ‘x explains y’ with ‘x’ and ‘y’ being singular terms referring to linguistic entities. Moreover, there is no straightforward way to transform “‘p’ explains ‘q’”, with ‘p’ and ‘q’ referring to linguistic items, into ‘because’-statements – the starting point for explanations according to the DN-model. “‘p’ because ‘q’” isn’t a well formed sentence. It is sentences which are used rather than mentioned which may flank
‘because’, or which form answers to the corresponding ‘why’-questions. Thus, it is hard to see how to give a coherent picture of direct theory-explanation, because this would violate the logic of ‘because’ and ‘explains’.

Now, assume that the logical problem could be solved, such that we actually get some sort of statement explanation. Just assume that ‘there was an earthquake’ explains ‘some windows shattered’. In this case, the explanation relation holds because these statements state facts about events which stand in an explanation relation to each other. Thus, the statement-explanation is derivative upon some other explanatory link. Here is why: In our example, the explanation is appropriate if there is a causal connection between there being an earthquake and the windows’ shattering. It is a causal explanation. In virtue of the fact that these sentences express what they express, we can say that ‘there was an earthquake’ explains ‘some windows shattered’ (if this is sound at all). Similarly for theories: no theory T (a syntactic representational pattern) explains any other theory T*, unless (and if so: in virtue of the fact that) what T states explains what T* states. Thus, direct theory-explanation (as opposed to indirect theory-explanation) is a model we are better off without: Even if we can make grammatical sense of direct theory-explanation, this sort of explanation would be derivative upon explanatory links between what theories state.

Fortunately, and, maybe, surprisingly, this is in accordance with the illustrations Nagel gave for theory-explanations. Although Nagel never explained why he believed reduction to consist of explanation of one theory by another, he gave an example which illustrates his idea of how the explanatory link is to be conceived of: The reduction of (a theory of) headaches. Nagel writes that when:
This passage contains most of the aspects directly relevant to reduction as explanation, directness and indirectness. If one theory is reducible to another theory, the reducing theory has the resources to explain the occurrence of the phenomena the reduced theory deals with. This is obviously a case of indirect reduction. Interestingly, reduction as phenomena explanation occurred in the writings of Nagel before he developed his model of reduction as a specific sort of theory explanation: In “The logic of Reduction in the Sciences” (Nagel 1935) he describes reduction as explanation and gives a characterization of explanation in terms of constitution-relations. It seems sound to assume that this is the sort of explanation Nagel tried to model when later introducing his notion of reduction as theory explanation (we will turn back to this issue in section 3). We thus get:

\(d.*\) Reduction is not direct (in the sense that it just is not a case of theory-explanation) – it goes together with explanations of the phenomena of the reduced theory by the reducing theory.

Now, describing the Nagel-model as being concerned with explanation, we face another uncertainty: Does this amount to the fact that Nagelian reduction should be regarded as purely epistemological? This seems to contradict the ontological interpretation of bridge-laws given above (identity and co-extensionality are concerned with a language’s ontology). However, maybe there is independent reason to describe Nagelian reduction as purely epistemological. So, let us consider the next point.
Here is the fifth feature of the official Nagel-model: *e. The Nagel-model is an epistemological model of reduction.*

### 2.5 Reduction as an epistemological issue

Why should we regard the Nagel-model as an epistemological model? Or, first of all, what is it to be an epistemological model? In one sense of the term, *epistemological reduction* is associated with a specific reductionist position, namely the position according to which some sort of reduction from one scientific level to another scientific level can actually be carried out. This, however, is not what Sarkar, who stresses this point (Sarkar 1992), has in mind. Rather, it is a question regarding the conceptual resources used to characterize the model, namely, whether or not the model is cashed out in purely epistemological terminology. And Sarkar seems to take this for granted, because it is cashed out in terms of *explanation* (recall his slogan: “explanatory reductionism is epistemological reductionism” (Sarkar 1992, 171)).

The problem with this slogan can roughly be described as follows: Indeed, explanations play a cognitive role; they are epistemologically relevant. However, this is not to say that they do not comprise ontological commitments, or that the notion of explanation is to be cashed out in purely epistemological terms. Consider an analogy: The concept of knowledge surely comprises an epistemological element. However, it also comprises the concept of truth, which is *not* an epistemological notion. Therefore, the fact that a given concept plays a role in describing epistemological issues does not license the claim that it is purely epistemological in nature. Quite the contrary is the case for Nagel-reduction; it
clearly commits us to strong metaphysical claims. It is explanation which is based on cross-theoretic ontological links. Obviously, Sarkar’s claim is not sensitive to this point.

As pointed out above, there are two variants of this claim: First, the model is judged epistemological because it is concerned with theories (Hoynigen-Huene 1989; Silberstein 2002, Sarkar 1998). It has been argued that this point is to be handled with care. But even if we allow for Nagelian reduction in case of fragments of ordinary discourse (and, thus, weaken the notion of a theory) we would presumably still have a case of epistemological reduction according to a similar criterion; it would still be a relation which holds between representational items. Since bridge-laws should, however, be interpreted ontologically (and even the explanation-relation should be conceived of as being concerned with theories in a derivative sense) the relevance of the distinction between ontological and epistemological reduction seems to break down. The model is (by definition?) epistemological because it describes reduction as a theory-relation. But at the same time, it should be regarded as being concerned with ontological issues – otherwise, we were unable to make sense of explanation of phenomena as well as of the bridge-principles which state ontological links. This latter point is also relevant for the assumption that according to Nagel, bridge-laws are purely epistemological in nature (Fazekas 2009; Klein 2009): As has been pointed out, Nagel believed bridge-laws to state identities or relations among extensions – and, hence, cannot be characterized in purely epistemological terms. Thus, even if the real Nagel-model is epistemological qua being (amongst other things) concerned with representational items, or qua being tied to explanation, or qua being such that its bridge-laws can be partly characterized in epistemological terminology, it should be obvious that in other respects, it is ontological.
Thus, I assume that the characterization of the model as being epistemological in contrast to being ontological is misguided.

In this sense:

\textit{e.} The Nagel-model is not an epistemological model of reduction.

We have thus finished the discussion of the official Nagel-model. Let us assemble the main features of the real Nagel-model:

\textit{According to the real Nagel-model,}

\textit{a.} reduction is a relation holding among a great variety of scientific representational devices, amongst which theories play an important epistemological role.

\textit{b.} interesting reductions are explanations which consist in deductions which are carried out with the help of bridge laws and they have to obey (some of) the relevant non-formal criteria (unification, appropriateness of reducing theory and bridge-laws, and, if possible, correction should be involved in reduction).

\textit{c.} Bridge-laws are to be regarded as stating ontological links (identities or relations among extensions) a posteriori.

\textit{d.} reduction is not direct – it goes together with explanations of the phenomena of the reduced theory by the reducing theory.

\textit{e.} reduction is not a purely epistemological issue.

It can now be shown that this model does not face most of the problems the official Nagel-model faces.
3. A stronger model of Reduction (Conclusion)

The most prominent objections to Nagel’s model of reduction have been summarized in the introduction. Two of them have already been shown to be misguided: that the model is not liberal enough concerning the relata of the reduction relation because it allows for theory reduction only, and that the model describes reduction in terms of direct theory-explanation, whereas an appropriate model of reduction should shape the notion in terms of indirect theory-explanation. These objections are clearly misguided when used to argue against the real Nagel-model. Moreover, worries like the one that any inconsistent theory would reduce any theory, or that any theory would reduce to itself could be addressed in the spirit of Nagel’s non-formal conditions – once we stipulate that the reducing theory should be well established, and that, say, bridge-laws are crucially used in the derivation, these formal worries could easily be blocked. As has been pointed out above (and has been argued by Putnam (1965) and Dizadji-Bahmani et al. (2010)) the Nagel model could easily be altered to cover cases of reductions involving correction. Thus, the criticisms of Feyerabend and others are, in this respect, misguided.

The syntactic view on theories Nagel embraced gives rise to an additional (alleged) problem; namely, that since the syntactic view is just misguided, the Nagel-model is fallacious (see, for example, Suppe 2000, 109). Let us just assume that any purely syntactic view on reduction is in fact problematic. Would this affect, without further ado, the real Nagel-model? It does not. Bridge-laws are to be interpreted semantically. They state identities or nomologically necessary relations among extensions, and these

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6 One might wonder what such a “purely syntactic” view would consist in. Even Hempel characterizes the linguistic version of reduction (as opposed to the ontological version) in semantic terminology, referring to conceptual differences between ways of presentation by different expressions (Hempel 1969).
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statements are not analytic. Stating something and being analytic are semantic categories. Therefore, reduction according to Nagel should be conceived of as holding between syntactic entities in virtue of these entities’ semantic properties. Thus, a semantic characterization of theories (or a theory’s laws) as well as of bridge-laws is crucial for an appropriate understanding of the real Nagel-model.

A closely related problem for the official Nagel-model is this: it seems designed to instantiate the DN-model of explanation. The DN-model, however, has been criticized for a number of reasons (see, for example, Salmon 1989). As has been pointed out above, the idea that reduction is concerned with explanation is independent of the assumption that reduction is concerned with theory-explanation. It is a sort of explanation which is exemplified by the explanation of the occurrence of headaches in terms of its physiological or chemical conditions. This interpretation does not commit Nagel to the claim that reduction has to be spelled out in terms of the DN-model. He gave no reason for why to conceive of theory reduction as theory explanation, he gave examples of reductive explanations which do not instantiate theory-explanation, and he described reduction as being indirect. So, the real Nagel-model, which takes these insights into account, does not face problems the DN-model of explanation faces. This becomes even more transparent once we try to answer the question of why Nagel shaped his model of reduction in terms of the DN-model.

Conceiving of reduction as derivation was common among early positivists. For Carnap (1934) and Neurath (1959), reduction was a matter of conceptual analysis (or at least: translation), and, therefore, a matter of derivation. Within this framework, inspired by classical interpretations of reduction, Nagel developed his model. Nagel altered the definition, introducing bridge-laws which are not to be regarded as mimicking conceptual
analyses. This seems to be the main point of his discussion of bridge-laws, and this is to be regarded as the most important amendment the view on reduction has undergone due to Nagel. Now, I tend to think that Nagel believed reduction to be explanation of theories by theories because he believed reduction to be concerned with explanation (and rightly so, as the case of headaches as well as his first paper on reduction (Nagel 1935) suggest) and he believed reduction to be derivation plus bridge-laws. He believed reduction to be derivation plus bridge-laws because this was the common way to conceive of reduction. If this is correct, then explanation as a relation between theories enters the game because Nagel believed, in addition, in the DN-model – a model which is instantiated by reduction as derivation plus bridge-laws.\(^7\) We have no independent reason to assume that reduction is explanation of one theory in terms of the other, and Nagel does not give us one. So, a lot of what has been discussed as lying at the heart of the Nagel-model in the literature (by Nagel himself and by others) is, maybe, merely the accidental (and highly misleading) outcome of Nagel’s philosophical background.

I would like to express my gratitude to Albert Newen, Stephan Hartmann, two anonymous reviewers and Dan Brooks for their suggestions and comments on earlier drafts of this paper. Moreover, I would like to thank the groups at the Philosophy Department of Ruhr-University of Bochum and at the Tilburg Center for Logic and Philosophy of Science, for stimulating discussions of related topics.

\(^7\) Nagel suggests this himself, writing that “[i]t is safe to say that [reductions] are commonly taken to be explanations […] In consequence, I will assume, that like all scientific explanations in general, every reduction can be construed as a series of statements, one of which is the conclusion […], while the others are the premises […]” (Nagel 1970, 119).
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